UWThPh-1996-26
FISIST/3-96/CFIF
February 1, 2008

On the Determination of Δm and $\Delta\Gamma$ in Tagged $D^0/\bar{D}^0$ Decays

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Abstract

We consider the time dependence of the decays of tagged D^0 and $\bar{D}^0$ into CP-conjugate final states f and $\bar{f}$, or into a CP eigenstate F. We expand each decay width as $\exp(-\Gamma t)$ times a series in Γt, where Γ is the average decay width of the mass eigenstates D_H and D_L, and examine the first three terms of the series. We show that experimental information on the coefficients of these terms allows in principle to compute all the relevant mixing parameters. In particular, depending on CP violation or conservation, we discuss the different possibilities to extract Δm and $\Delta\Gamma$, i.e., the mass and decay-width differences of the mass eigenstates. We also comment on consistency conditions among the coefficients of the various decay-width expansions.

In this paper we consider the system of the charmed neutral mesons D^0 and $\bar{D}^0$. The mass eigenstates of that system are given by

$$\begin{aligned} |D_H\rangle &= p|D^0\rangle + q|\bar{D}^0\rangle\,, \\ |D_L\rangle &= p|D^0\rangle - q|\bar{D}^0\rangle\,, \end{aligned} \tag{1}$$

where the index H refers to heavy and L to light. We denote by $\Gamma = (\Gamma_H + \Gamma_L)/2$ the average decay width of D_H and D_L. The standard model (SM) predicts very small parameters

$$x = \frac{m_H - m_L}{\Gamma} \quad \text{and} \quad y = \frac{\Gamma_H - \Gamma_L}{2\Gamma}\,. \tag{2}$$

The parameter x is positive by definition. Experimentally, it is already known that $(x^2 + y^2) \lesssim 10^{-2}$ [2]. Therefore, even if physics beyond the SM is very important in the D^0–$\bar{D}^0$ system, at most the onset of oscillations can be discovered [3]. This fact is exploited in discussions of tagged $D^0/\bar{D}^0$ decays with decay-time information, where in the time-dependent decay widths one performs an expansion [3, 4, 5] with respect to the quantity $(x - iy)\Gamma t$, which is small as long as Γt is of order one. It is reasonable to truncate the expansion at order $(\Gamma t)^2$. There is some hope that in future experiments the coefficients of this expansion will be measured. In this paper we propose to use these coefficients to get information on x and y and also on CP violation. We show that if one measures them up to order $(\Gamma t)^2$ in all four decay widths $\Gamma(D^0(t)/\bar{D}^0(t) \to f/\bar{f})$ of a given Cabibbo-allowed/doubly-Cabibbo-suppressed pair $f/\bar{f}$ of CP-conjugate final states one can unambiguously extract x, y, and also information on CP violation in mixing. The simplest such final state would be $f = K^-\pi^+$. We also include a discussion of final CP eigenstates F like $F = K^+K^-$.

To study phenomenologically the time dependence of the widths of tagged D^0 and $\bar{D}^0$ decays into f and $\bar{f}$ the following four amplitudes are relevant:

$$\mathcal{A}(D^0 \to f) \equiv A,\ \mathcal{A}(\bar{D}^0 \to \bar{f}) \equiv \bar{A},\ \mathcal{A}(\bar{D}^0 \to f) \equiv B,\ \mathcal{A}(D^0 \to \bar{f}) \equiv \bar{B}. \tag{3}$$

As mentioned above the starting point of our analysis is an expansion of the time-dependent $D^0/\bar{D}^0$ decay widths with respect to $(x - iy)\Gamma t$. The first three terms in this expansion are given by

$$\Gamma(\overset{(-)}{D^0}(t) \to \overset{(-)}{f}) = \mathrm{e}^{-\Gamma t}[\overset{(-)}{a}(\overset{(-)}{f}) + \overset{(-)}{b}(\overset{(-)}{f})\,\Gamma t + \overset{(-)}{c}(\overset{(-)}{f})\,(\Gamma t)^2 + \ldots] \tag{4}$$

with the coefficients

$$\begin{aligned} a(f) &= |A|^2, \quad & b(f) &= \mathrm{Im}[(x - iy)\tfrac{q}{p}A^* B], \\ \bar{a}(f) &= |B|^2, \quad & \bar{b}(f) &= \mathrm{Im}[(x - iy)\tfrac{p}{q}B^* A], \\ a(\bar{f}) &= |\bar{B}|^2, \quad & b(\bar{f}) &= \mathrm{Im}[(x - iy)\tfrac{q}{p}\bar{B}^* \bar{A}], \\ \bar{a}(\bar{f}) &= |\bar{A}|^2, \quad & \bar{b}(\bar{f}) &= \mathrm{Im}[(x - iy)\tfrac{p}{q}\bar{A}^* \bar{B}], \end{aligned}$$

$$
\begin{aligned}
c(f) &= \tfrac{1}{4}[\eta^2|B|^2(x^2+y^2) - |A|^2(x^2-y^2)],\\
\bar{c}(f) &= \tfrac{1}{4}[\tfrac{1}{\eta^2}|A|^2(x^2+y^2) - |B|^2(x^2-y^2)],\\
c(\bar{f}) &= \tfrac{1}{4}[\eta^2|\bar{A}|^2(x^2+y^2) - |\bar{B}|^2(x^2-y^2)],\\
\bar{c}(\bar{f}) &= \tfrac{1}{4}[\tfrac{1}{\eta^2}|\bar{B}|^2(x^2+y^2) - |\bar{A}|^2(x^2-y^2)],
\end{aligned}
\tag{5}
$$

where $\eta \equiv |q/p|$ parametrizes T and CP violation in D^0–$\bar{D}^0$ mixing. We assume that Γ and the coefficients in eqs. (5) can be obtained from a fit of eq. (4) to the $D^0/\bar{D}^0$ decays with decay-time information.

Let us first assume that we have the coefficients $\overset{(-)}{a}(\overset{(-)}{f})$ and $\overset{(-)}{c}(\overset{(-)}{f})$ at our disposal. This allows us to compute x and y as functions of these coefficients and of η:

$$
\begin{aligned}
x^2 &= 2\,\frac{\eta^2\bar{c}(f) - c(f)}{a(f) - \eta^2\bar{a}(f)} = 2\,\frac{\eta^2\bar{c}(\bar{f}) - c(\bar{f})}{a(\bar{f}) - \eta^2\bar{a}(\bar{f})}\,,\\
y^2 &= 2\,\frac{\eta^2\bar{c}(f) + c(f)}{a(f) + \eta^2\bar{a}(f)} = 2\,\frac{\eta^2\bar{c}(\bar{f}) + c(\bar{f})}{a(\bar{f}) + \eta^2\bar{a}(\bar{f})}\,.
\end{aligned}
\tag{6}
$$

This means that given η, then x and y can be obtained from considering tagged $D^0/\bar{D}^0$ decays into either f or $\bar{f}$. If the full information is used the consistency condition

$$
c(f)\bar{a}(\bar{f}) + \bar{c}(f)a(\bar{f}) = \bar{c}(\bar{f})a(f) + c(\bar{f})\bar{a}(f) \tag{7}
$$

is obtained. Similarly, η can be extracted by

$$
\eta^4 = \frac{c(\bar{f})a(f) - c(f)a(\bar{f})}{\bar{c}(f)\bar{a}(\bar{f}) - \bar{c}(\bar{f})\bar{a}(f)}\,. \tag{8}
$$

It is important to note that the determination of η depends not only on the observation of the shape of the decay curves of $D^0(t)$ and $\bar{D}^0(t)$, but also on the *relative normalization* of these decay curves. This means that it is not just the ratios $c(\bar{f}) : c(f) : a(\bar{f}) : a(f)$ and $\bar{c}(f) : \bar{c}(\bar{f}) : \bar{a}(f) : \bar{a}(\bar{f})$ which are important; for the computation of η the relative normalization $a(f) : \bar{a}(f)$ is fundamental too. This is a point relevant for the observation of T violation in the mixing of any neutral-meson system [6]. It is easy to check by inserting eq. (8) into eqs. (6) that the knowledge of this relative normalization is not necessary for the determination of x and y.

Let us now assume instead that the coefficients $\overset{(-)}{a}(\overset{(-)}{f})$ and $\overset{(-)}{b}(\overset{(-)}{f})$ are available. Then, we can make a simple parameter counting. If we add η to those eight coefficients, there is a total of nine measurements. On the other hand, only seven rephasing–invariant quantities can be formed from q/p and from the amplitudes in eq. (3) [7]. Therefore, there are two consistency conditions involving the parameters x and y, which are thus calculable. These conditions are

$$
\frac{1}{x^2}\left[b(f) - \eta^2\bar{b}(f)\right]^2 + \frac{1}{y^2}\left[b(f) + \eta^2\bar{b}(f)\right]^2 = 4a(f)\bar{a}(f)\eta^2 \tag{9}
$$

and

$$\frac{1}{x^2}\left[b(\bar f)-\eta^2\bar b(\bar f)\right]^2+\frac{1}{y^2}\left[b(\bar f)+\eta^2\bar b(\bar f)\right]^2=4\bar a(\bar f)a(\bar f)\eta^2, \tag{10}$$

which are equations for ellipses in the variables $1/x$ and $1/y$. Eq. (10) is obtained from eq. (9) by simply replacing f by $\bar f$. ¿From the system of eqs. (9) and (10) one can find x (which is positive by definition) and $|y|$. This method for the determination of the mass and decay-width differences only works, however, if η is already known. This could be from eq. (8), for instance. If we combine eqs. (9) and (10) and eqs. (6) we get two further consistency conditions analogous to eq. (7).

If CP is conserved we have $\eta = 1$ and the relations

$$a(f)=\bar a(\bar f),\ \bar a(f)=a(\bar f),\ b(f)=\bar b(\bar f),\ b(\bar f)=\bar b(f),\ c(f)=\bar c(\bar f),\ c(\bar f)=\bar c(f) \tag{11}$$

are fulfilled. Then the two ellipses coincide and x and y cannot be disentangled. Therefore, eqs. (9) and (10) constitute another version of the observation that large CP violation can be quite helpful in getting a hold on mixing in the D^0–$\bar D^0$ system [5]. Nevertheless, even in the case of CP conservation one could have an interesting restriction in the $x-y$ plane, and consequently lower bounds on x and y.

The previous method using $\overset{(-)}{c}$ $(\overset{(-)}{f})$ does not suffer from the above problem, i.e., x and y can be determined even when CP is conserved. However, in that case the two ways (see eqs. (6)) of obtaining x and y via the final states f and $\bar f$ coincide.

If CP is conserved the constraint eq. (7) is automatically satisfied because of the equalities in eqs. (11). However, CP conservation introduces an additional constraint among the coefficients $\overset{(-)}{a}(f)$, $\overset{(-)}{b}(f)$ and $\overset{(-)}{c}(f)$, (or, analogously, among the coefficients $\overset{(-)}{a}(\bar f)$, $\overset{(-)}{b}(\bar f)$ and $\overset{(-)}{c}(\bar f)$) which reads

$$4a\bar a(\bar c^2-c^2)=(b^2+\bar b^2)(a\bar c-\bar a c)+2b\bar b(\bar a\bar c-ac). \tag{12}$$

We have left out the labels f or $\bar f$ for simplicity of notation.

We now proceed to an analogous discussion of $D^0/\bar D^0$ decays into a CP eigenstate F. In this case the expansion of the decay widths reads

$$\Gamma(\overset{(-)}{D^0}(t)\to F)=\mathrm{e}^{-\Gamma t}[\overset{(-)}{a}(F)+\overset{(-)}{b}(F)\,\Gamma t+\overset{(-)}{c}(F)\,(\Gamma t)^2+\ldots] \tag{13}$$

with

$$\begin{aligned}
a(F)&=|A|^2, \quad b(F)=\mathrm{Im}[(x-iy)\tfrac{q}{p}A^*\bar A],\\
\bar a(F)&=|\bar A|^2, \quad \bar b(F)=\mathrm{Im}[(x-iy)\tfrac{p}{q}A\bar A^*],\\
c(F)&=\tfrac14[\eta^2|\bar A|^2(x^2+y^2)-|A|^2(x^2-y^2)],\\
\bar c(F)&=\tfrac14[\tfrac{1}{\eta^2}|A|^2(x^2+y^2)-|\bar A|^2(x^2-y^2)].
\end{aligned} \tag{14}$$

In the following we shall leave out the label F in the coefficients. The expansion of eq. (13) supplies us with six experimental quantities. This number has to be confronted with the

number of independent physical quantities constructed from $A = \bar{B}$, $\bar{A} = B$ and q/p, which is now four [7]. Therefore, at least in principle, one can obtain x and y from such measurements. Analogous to eqs. (6) we find

$$x^2 = 2\,\frac{\eta^2 \bar{c} - c}{a - \eta^2 \bar{a}}\,, \quad y^2 = 2\,\frac{\eta^2 \bar{c} + c}{a + \eta^2 \bar{a}}\,. \tag{15}$$

Of course, here there is no consistency condition such as eq. (7). Also, η cannot be obtained from the coefficients $\overset{(-)}{a}$ and $\overset{(-)}{c}$ alone. On the other hand, from b and $\bar{b}$ we get a single ellipse

$$\frac{1}{x^2}(b - \eta^2 \bar{b})^2 + \frac{1}{y^2}(b + \eta^2 \bar{b})^2 = 4 a \bar{a} \eta^2 \tag{16}$$

in the variables $1/x$ and $1/y$. Taking eqs. (15) and (16) together we can compute

$$\eta^4 = \frac{b^2(\bar{a}c - a\bar{c}) + 2b\bar{b}ac - 4a\bar{a}c^2}{\bar{b}^2(a\bar{c} - \bar{a}c) + 2b\bar{b}\bar{a}\bar{c} - 4a\bar{a}\bar{c}^2}\,, \tag{17}$$

and then from eqs. (15) the values of x and y can be obtained.

CP conservation is a much stronger restriction on the phenomenology of decays into CP eigenstates F than of decays into f and $\bar{f}$, because it gives not only

$$a = \bar{a},\ b = \bar{b},\ c = \bar{c}, \tag{18}$$

but also

$$|b| = |y| a \quad \text{and} \quad c = \frac{1}{2} y^2 a, \tag{19}$$

and therefore

$$2ac = b^2. \tag{20}$$

In this case the expressions for x and η are of the type $0/0$ and thus undefined. As a consequence, if CP violation in mixing and amplitudes is small, it will be difficult to constrain x and η from the decay of tagged $D^0/\bar{D}^0$ into CP eigenstates. However, eq. (19) shows that CP eigenstates are well suited to get information on y [8]. Using CP non-eigenstates one does not face the problem of undefined expressions in the case of CP conservation. The denominators in eqs. (6) and (8) are different from zero, because the amplitude B is doubly-Cabibbo-suppressed, i.e., $B \sim \lambda^2 A$ with $\lambda \approx 0.22$.

In conclusion, we have studied the conditions under which we can extract the mixing parameters x and y of the D^0–$\bar{D}^0$ system from the measurement of the first three terms in an expansion in $(x - iy)\Gamma t$ of the decay widths of tagged $D^0(t)$ and $\bar{D}^0(t)$. We found that this task can indeed be performed in several ways if one considers the decays into a pair of CP-conjugate final states. If one uses the coefficients of $(\Gamma t)^0$ and $(\Gamma t)^2$ it suffices to consider either f or $\bar{f}$ as a final state, provided the parameter η measuring CP violation in D^0–$\bar{D}^0$ mixing is determined from elsewhere. If the coefficients of all four decay widths are given then η can be determined as well and, in addition, a consistency

condition among the coefficients has to be fulfilled. Experimental knowledge of the coefficients of $(\Gamma t)^0$ and $(\Gamma t)^1$ for both final states f and $\bar{f}$ allows the extraction of y, but in order to obtain x, CP must be violated. We have also discussed the decays into a CP eigenstate. Again CP violation is necessary to determine the mixing parameters. In any case, in the framework discussed here, the determination of η seems to be the most difficult experimental problem. Needless to say, all the different methods we put forward to get information on D^0–$\bar{D}^0$ mixing can be combined in various ways. The experimental situation will pin down the most useful strategy.

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